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SOLID GEOMETRY

THE PERPENDICULAR DISTANCE FROM A POINT TO THE PLANE

MATHEMATICS

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THE PERPENDICULAR DISTANCE FROM A POINT TO THE PLANE

THEOREM : The perpendicular distance from a point A (x_1, y_1, z_1) to the plane **ax +by +cz + d = 0** i.e the length of the perpendicular from a point A(x_1, y_1, z_1) to the plane ax +by +cz + d = 0 is $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

PROOF : Let the equation of the plane be Π = ax +by +cz + d=0 ….. (1)
Let A = A(x_1, y_1, z_1) be the point (A ∉ Π) from which the perpendicular drawn to the plane Π meets it in C.

Let the normal form of plane Π be lx +my + nz = p…… (2)
The equation of the plane parallel to eq (2) and passing through the point A be lx+my+nz= p_1 … (3) when $lx_1+my_1+nz_1=p_1$…… (4)

Problems :

1.Find the perpendicular distance from the point (1,-2,8) to the plane 2x-3y+6z = 63.

Solution: Given plane is 2x-3y+6z -63=0

Here a =2,b = -3,c = 6 d = -63

Given point A (1,-2,8) = A(x_1, y_1, z_1)

The perpendicular distance from a point A (x_1, y_1, z_1) to the plane ax +by +cz +d = 0 is $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

The perpendicular distance from the point A(1,-2,8) to the plane 2x-3y+6z = 63

$$= \left|\frac{2(1)-3(-2)+6(8)-63}{\sqrt{(2^2)+(-3^2)+(6^2)}}\right|$$

$$= \left|\frac{2+6+48-63}{\sqrt{4+9+36}}\right|$$

$$= \left|\frac{7}{\sqrt{49}}\right|$$

$$= \left|\frac{7}{7}\right|$$

= 1 unit

$\therefore$ The perpendicular distance from A(1,-2,8) to the plane 2x-3y+6z = 63 is 1 unit.

2.Find the perpendicular distance from the point (3,-4,1) to the plane x+y-z+7=0.

Solution : Given plane is x+y-z+7=0 :Here a = 1,b = 1,c = -1,d=7

Given point A(3,-4,1) = A(x_1, y_1, z_1)

The perpendicular distance from a point A (x_1, y_1, z_1) to the plane

ax + by + cz + d = 0 is $\left| \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}} \right|$

The perpendicular distance from the point A(3,-4,1) to the plane x+y-z+7=0

$= \left|\frac{1(3)+1(-4)-1(1)+7}{\sqrt{1^2+1^2+(-1)^2}}\right|$

$= \left|\frac{3-4-1+7}{\sqrt{1+1+1}}\right|$

$= \left|\frac{5}{\sqrt{3}}\right|$ units

∴The perpendicular distance from the point A(3,-4,1) to the plane x+y-z+7=0 is

$\left| \frac{5}{\sqrt{3}} \right|$ **Units**

NOTE : The perpendicular distance from the origin to the plane ax +by +cz +d = 0 is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$

Problem :Find the perpendicular distance from the origin to the plane 3x+4y-12z+65=0.

Solution: Given plane is 3x+4y-12z+65=0. Here a= 3,b= 4,c= -12,d= 65

Given point is origin =(0,0,0)

The perpendicular distance from the origin to the plane ax +by +cz + d = 0 is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$

$$= \frac{|65|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$$

$$= \frac{|65|}{\sqrt{9+16+144}}$$

$$= \frac{|65|}{\sqrt{169}}$$

$$= \frac{65}{13} = 5\text{units}$$

The perpendicular distance from the origin to the plane 3x+4y-12z+65=0 is 5units.

DISTANCE BETWEEN THE PARALLEL PLANES:

THEOREM : The distance between the parallel planes ax + by +cz +d_1=0, ax +by +cz +d_2=0 is $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

PROOF : The equation to the planes are ax +by +cz +$d_1 = 0$,

And $ax + by + cz + d_2 = 0 ..(1)$

The dc's of the normal to the planes are $\left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$

Now p_1 and p_2 be the perpendicular distances to the planes from origin

$\Rightarrow p_1 = \frac{-d_1}{\sqrt{a^2+b^2+c^2}}$, $p_2 = \frac{-d_2}{\sqrt{a^2+b^2+c^2}}$

$\Rightarrow$The distance between the parallel planes $=|p_1 - p_2|$

$$= \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

$\therefore$ The distance between the parallel planes is $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

Hence the theorem.

PROBLEMS:

1.Find the distance between the parallel planes 12x-3y+4z-7 = 0 and 12x-3y+4z+6 =0

SOLUTION:

The given planes are planes 12x-3y+4z-7 = 0 and 12x-3y+4z+6 =0

Here a = 12,b = -3,c = 4 and $d_1 = -7, d_2 = 6$

The distance between the parallel planes is $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

The distance between the given planes = $\frac{|-7-6|}{\sqrt{12^2+(-3)^2+4^2}}$

$= \frac{13}{\sqrt{144+9+16}}$

$= \frac{13}{\sqrt{169}}$

$=\frac{13}{13}$

=1 unit

∴ The distance between the parallel planes is 1 unit

2.Prove that the distance between the parallel planes 2x-2y+z+3 =0 and 4x-4y +2z+5 =0 is 1/6

SOLUTION :

The given planes are 2x-2y+z+3 =0 and 4x-4y +2z+5 =0

$$\Rightarrow 2x-2y+z+\frac{5}{2} = 0$$

Here a = 2,b = -2,c = 1 and $d_1 = 3, d_2 = \frac{5}{2}$

The distance between the parallel planes = $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

$$= \frac{|3-\frac{5}{2}|}{\sqrt{2^2+-2^2+1^2}}$$

$$= \frac{|\frac{6-5}{2}|}{\sqrt{4+4+1}}$$

$$= \frac{\frac{1}{2}}{\sqrt{9}}$$

$$= \frac{1}{6}\, units$$

∴ The distance between the parallel planes is 1/6 units

Prescribed Text Book:

Analytical Solid Geometry by Shanti Narayan and P.K. Mittal, published by S. Chand & Company Ltd. 7th Edition.

Reference Books :

1.A text book of Mathematics for BA/B.Sc Vol 1, by V Krishna Murthy & Others, published by S. Chand & Company, New Delhi.

2.A text Book of Analytical Geometry of Three Dimensions, by P.K. Jain and Khaleel Ahmed, published by Wiley Eastern Ltd., 1999.

3.Co-ordinate Geometry of two and three dimensions by P. Balasubrahmanyam, K.Y. Subrahmanyam,

4.G.R. Venkataraman published by Tata-MC Gran-Hill Publishers Company Ltd., New Delhi.